International Journal of Thermophysics, Vol. 23, No. 4, July 2002 (© 2002)

Critical and Flash Points for Metastable Systems in a Mean-Field Approach[1]

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By using, as a model, an analytical equation of state which describes a system that can exist in a liquid or vapor phase, scaling properties regarding the critical temperature and the "flash" temperature were found. The "flash" temperature is defined as the highest temperature at which a self-bound system can exist in hydrostatic equilibrium. The "flash" temperature can provide an alternate natural dimension-dependent scale of temperature, other than the critical temperature.

KEY WORDS: equation of state; model; statistical mechanics; vapor-liquid equilibria.

We consider a simple equation of state (EOS) derived from a Skyrme-type interaction [1]. Such an interaction is short-ranged and typically used in nuclear physics. Without any loss of generality in our study, it is particularly well suited for our purposes since it allows an analytical derivation of the scaling properties:

$$p = -a_o \rho^2 + 2a_3 \rho^3 + \rho k_B T. \tag{1}$$

This equation shows explicitly a cubic dependence on the density ρ, in much the same way as for the van der Waals fluid. This form for the EOS is typical of a system that can exist in a liquid or vapor phase, and suggests the existence, at low density, of a line of first-order liquid-vapor phase

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[1] Paper presented at the Fourteenth Symposium on Thermophysical Properties, June 25–30, 2000, Boulder, Colorado, U.S.A.

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0195-928X/02/0700-0949/0

transition in a p versus T phase diagram, ending up at a critical point, where the transition is continuous. The temperature T_c associated with this critical point is an upper bound for the range of temperatures in which the two phases coexist. Many other models for nuclear matter have been studied [2–4], all of them exhibiting an EOS with the same van der Waals fluid-like behavior.

We are aware that the use of a mean-field approach (MFA) has as a drawback that the actual spatial dimension in which the system is embedded loses in part its specificity, as far as the critical behavior is concerned. The critical exponents have the same values in all dimensions, and the equations of state collapse into one single curve in the neighborhood of criticality, when rescaled with the critical parameters. For a simple van der Waals gas, this collapse occurs over the entire range of the thermodynamic variables, as can be trivially verified [5]. The same is true for the analytically soluble model with Skyrme-type interactions mentioned above.

Mean-field phase diagrams can be valuable, even below the upper critical dimension of a model, to explore its regions of metastability. As is well known from the study of fluids, a physical system can get trapped in a local minimum of its free energy, from which it escapes only after a finite time. In an MFA the positions of these local minima are bounded in the phase diagram by the spinodal curves, which lie inside the region of phase coexistence. One usually considers the isothermal, for quenches through processes at constant temperature, and adiabatic (or isoentropic) spinodals, which can be formally determined by the solutions of $\partial p/\partial \rho = 0$, keeping constant the appropriate thermodynamic variable. The processes of fragmentation and super-heating are associated with the regions $\partial p/\partial \rho < 0$ and $\partial p/\partial \rho > 0$, respectively. Since we will be addressing in this paper only isothermal quenches, these derivatives will be taken at constant temperature.

A particularly interesting region still inside the coexistence region is that in which a hydrostatic equilibrium ($p = 0$) is still possible and the nuclear matter incompressibility,

$$K(T) = \left.\frac{\partial p}{\partial \rho}\right|_{p=0} \tag{2}$$

can be calculated. This region is bounded by $0 < T < T_{fl}$ and $\rho_{fl} < \rho < \rho_0$, where the point (ρ_{fl}, T_{fl}) is obtained as the solution to $p = \partial p/\partial \rho = 0$ and is known as the “flash” point. This point represents the smallest density and the highest temperature at which a self-bound system can exist in hydrostatic equilibrium, and belongs by definition to a spinodal. It can thus provide an alternate natural dimension-dependent scale of temperatures, other than the critical temperature. It is worth mentioning at this point that critical behavior at spinodal points has been found in the study of mean-field versions of classical models in condensed matter, such as the inconspicuous Ising model [6].

Here we proceed to the discussion regarding scaling with critical and flash parameters through the simple analytical EOS, given by Eq. (1) [1]. Although derived for a $3D$ system [7], a straightforward reproduction of that reasoning can be used to show that it has the same functional form in every spatial dimension. The relation between its coefficients and those of the interaction potential, however, are dimension-dependent. These coefficients have dimensions $[a_0]=M^{-D+1}$ and $[a_3]=M^{1-2D}$. The density at saturation of the nuclear matter, defined as the largest density for which hydrostatic equilibrium is still possible, is obtained as a solution to $p(\rho_0)=0$, leading to $\rho_0=\frac{a_0}{4a_3}\,[1+\sqrt{1-(8a_3k_BT)/a_0^2}]$. The incompressibility (at nuclear saturation) is obtained from Eq. (2) yielding

$$K(T)=\frac{a_0^2}{4a_3}\left[1+\sqrt{1-\frac{8a_3k_BT}{a_0^2}}-\frac{8a_3k_BT}{a_0^2}\right] \tag{3}$$

and $K(0)=\frac{a_0^2}{2a_3}$. The reduced incompressibility can thus be written as

$$\frac{K(T)}{K(0)}=\frac{1}{2}\left[1+\sqrt{1-\frac{8a_3k_BT}{a_0^2}}-\frac{8a_3k_BT}{a_0^2}\right]. \tag{4}$$

We begin by deriving a law of corresponding states for this EOS by rescaling the thermodynamical variables with their critical values. The critical point at which the liquid-vapor coexistence phase disappears and matter starts to be described as a gas is obtained via

$$\left.\frac{\partial p}{\partial \rho}\right|_{\rho=\rho_c}=\left.\frac{\partial^2 p}{\partial \rho^2}\right|_{\rho=\rho_c}=0 \tag{5}$$

leading to

$$\rho_c=\frac{a_o}{6a_3},\qquad k_BT_c=\frac{a_0^2}{6a_3},\qquad p_c=\frac{a_0^3}{108a_3^2} \tag{6}$$

with

$$a_0=\frac{k_BT_c}{\rho_c},\qquad a_3=\frac{k_BT_c}{6\rho_c^2}. \tag{7}$$

Substituting the values of a_0 and a_3 into Eqs. (1) and (4) one obtains

$$p' = \rho'^3 - 3\rho'^2 + 3\rho' T' \tag{8}$$

and

$$\frac{K(T)}{K(0)} = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4}{3}T'} - \frac{4}{3}T'\right]. \tag{9}$$

where $p' = p/p_c$, $\rho' = \rho/\rho_c$, and $T' = T/T_c$.

In this particular case, $p_c/k_B T_c \rho_c = 1/3$, near the $3/8$ value obtained for the van der Waals gas. Equation (8) is an expression of a law of corresponding states valid across different spatial dimensions.

Now, let us show that a similar law can be obtained when the variables are rescaled through their "flash point" values. At this point,

$$\left.\frac{\partial p}{\partial \rho}\right|_{\rho = \rho_f} = p(\rho_f) = 0. \tag{10}$$

Imposing the above conditions on Eq. (1) we find

$$\rho_f = \frac{a_0}{4a_3}, \qquad k_B T_f = \frac{a_0^2}{8a_3}, \qquad p_f = 0 \tag{11}$$

with

$$a_0 = \frac{2k_B T_f}{\rho_f}, \qquad a_3 = \frac{k_B T_f}{2\rho_f^2}, \tag{12}$$

which when substituted back into Eqs. (1) and (4) lead to

$$p^* = \rho^{*3} - 2\rho^{*2} + \rho^* T^*, \tag{13}$$

and

$$\frac{K(T)}{K(0)} = \frac{1}{2}[1 + \sqrt{1 - T^*} - T^*]. \tag{14}$$

where

$$p^* = p/k_B \rho_f T_f, \qquad \rho^* = \rho/\rho_f, \qquad T^* = T/T_f. \tag{15}$$

Here, p^* does not scale with the "flash" parameter p_f which is identically zero by construction, but with $k_B \rho_f T_f$ instead.

We can see in this case that $T_c/T_f = \frac{4}{3}$. Equations (9) and (14) relate the incompressibility curves in terms of different criticalities. Therefore, we found that the flash temperature can provide an alternate natural dimension-dependent scale of temperature, other than the critical temperature.

ACKNOWLEDGEMENTS

A.D. acknowledges partial financial support from the Brazilian agency Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq); J.S.S.M. received support from DOE Grant DE-FG03-95ER14499. The authors thank two anonymous referees for their relevant suggestions.

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